Int J Thermophys (2009) 30:2078–2097
DOI 10.1007/s10765-009-0686-z

Effect of Rotation in a Generalized Thermoelastic Medium with Hydrostatic Initial Stress Subjected to Ramp-Type Heating and Loading

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Received: 9 January 2009 / Accepted: 15 November 2009 / Published online: 29 November 2009

Abstract The present problem is concerned with the study of deformation of a rotating generalized thermoelastic medium with a hydrostatic initial stress. A linear temperature ramping function is used to more realistically model thermal loading of the half-space surface. The components of displacement, force stress, and temperature distribution are obtained in the Laplace and Fourier domains by applying integral transforms. The general solution obtained is applied to a specific problem of a half-space subjected to ramp-type heating and loading. These components are then obtained in the physical domain by applying a numerical inversion method. Some particular cases are also discussed in the context of the problem. The results are also presented graphically to show the effect of rotation and hydrostatic initial stress.

Keywords Generalized thermoelasticity · Hydrostatic initial stress · Laplace and Fourier transforms · Ramping parameter · Rotation · Temperature distribution

List of symbols

λ, μ	Lame's constants
ρ	Density
$\vec{u}$	Displacement vector
t_{ij}	Stress tensor

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τ_0, ϑ_0	Thermal relaxation times
$\upsilon = (3\lambda + 2\mu)\,\alpha_t$	Linear thermal expansion
$e = \mathrm{div}\vec{u}$	Dilatation
$K^\bullet$	Coefficient of thermal conductivity
C_E	Specific heat
t_0	Ramping parameter
p	Initial pressure
η	Initial stress parameter
E	Young's modulus
σ	Poisson ratio

1 Introduction

Generalized thermoelasticity theories have been developed with the objective of removing the paradox of infinite speed of heat propagation inherent in the conventional coupled dynamical theory of thermoelasticity in which the parabolic type heat conduction equation is based on Fourier's law of heat conduction. This newly emerged theory, which admits a finite speed of heat propagation, is now referred to as the hyperbolic thermoelasticity theory [1], as the heat equation for a rigid conductor is a hyperbolic-type differential equation.

There are two important generalized theories of thermoelasticity. The first is due to Lord and Shulman [2]. The second generalization to the coupled theory of thermoelasticity is what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity. Muller [3], in a review of the thermodynamics of a thermoelastic solid, proposed an entropy production inequality, with the help of which he consider restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws [4]. Green and Lindsay (G–L) [5] obtained another version of the constitutive equations. These equations were also obtained independently and more explicitly by Suhubi [6]. This theory contains two constants that act as relaxation times and modify all the equations of the coupled theory, and not only the heat equations. The classical Fourier law was violated if the medium under consideration has a center of symmetry.

Barber and Martin-Moran [7] discussed Green's functions for transient thermoelastic contact problems for the half-plane. Barber [8] studied thermoelastic displacements and stresses due to a heat source moving over the surface of a half-plane. Sherief [9] obtained components of stress and temperature distributions in a thermoelastic medium due to a continuous source. Dhaliwal et al. [10] investigated thermoelastic interactions caused by a continuous line heat source in a homogeneous isotropic unbounded solid. Chandrasekharaiah and Srinath [11] studied thermoelastic interactions due to a continuous point heat source in a homogeneous and isotropic unbounded body. Sharma et al. [12] investigated the disturbance due to a time-harmonic normal point load in a homogeneous isotropic thermoelastic half-space. Sharma and Chauhan [13] discussed mechanical and thermal sources in a generalized thermoelastic half-space. Sharma et al. [14] investigated the steady-state response of an applied load moving with constant speed for an infinite long time over the top surface of a homogeneous thermoelastic layer lying over an infinite half-space. Recently, Deswal and Choudhary [15] studied a two-dimensional problem due to a moving load in a generalized thermoelastic solid with diffusion.

The development of initial stresses in the medium is due to many reasons, for example, resulting from differences of temperature, process of quenching, shot pinning and cold working, slow process of creep, differential external forces, gravity variations, etc. The earth is assumed to be under high initial stresses. It is, therefore, of much interest to study the influence of these stresses on the propagation of stress waves. Biot [16] showed the acoustic propagation under initial stresses, which is fundamentally different from that under a stress-free state. He has obtained the velocities of longitudinal and transverse waves along the co-ordinate axis only.

The wave propagation in solids under initial stresses has been studied by many authors for various models. The study of reflection and refraction phenomena of plane waves in an unbounded medium under initial stresses is due to Chattopadhyay et al. [17], Sidhu and Singh [18], and Dey et al. [19]. Montanaro [20] investigated the isotropic linear thermoelasticity with a hydrostatic initial stress. Singh et al. [21], Singh [22], and Othman and Song [23] studied the reflection of thermoelastic waves from a free surface under a hydrostatic initial stress in the context of different theories of generalized thermoelasticity.

Misra et al. [24] studied the magneto-thermoelastic interaction in an aeolotropic solid cylinder subject to ramp-type heating. Misra et al. [25] studied thermoelastic interactions in an elastic half-space subjected to ramp-type heating. Youssef [26] constructed a model of the dependence of the modulus of elasticity and the thermal conductivity on the reference temperature and solved the problem of an infinite material with a spherical cavity. Youssef [27] studied the two-dimensional generalized thermoelasticity problem for a half-space subjected to ramp-type heating. Youssef [28] studied the problem of a generalized thermoelastic infinite medium with a cylindrical cavity subjected to ramp-type heating and loading. Youssef and Al-Harby [29] investigated a state-space approach of two-temperature generalized thermoelasticity of an infinite body with a spherical cavity subjected to different types of thermal loading. Youssef [30] investigated the two-dimensional problem of a two-temperature generalized thermoelastic half-space subjected to ramp-type heating.

Some researchers in the past have investigated a different problem of rotating media. Chand et al. [31] presented an investigation on the distribution of deformation, stresses, and magnetic field in a uniformly rotating homogeneous isotropic, thermally and electrically conducting elastic half-space. Numerous authors [32–34] studied the effect of rotation on elastic waves. Roychoudhuri and Mukhopadhyay [35] studied the effect of rotation and relaxation times on plane waves in generalized thermo-viscoelasticity. Ting [36] investigated the interfacial waves in a rotating anisotropic elastic half-space. Sharma and his co-workers [37–40] discussed the effect of rotation on a different type of waves propagating in a thermoelastic medium. Othman and Song [41,42] presented the effect of rotation in magneto-thermoelastic medium. Ailawalia and Narah [43] discussed the effect of rotation due to a moving load at the interface of an elastic half-space and a generalized thermoelastic half-space.

In the present investigation, we have obtained the expressions for displacement, force stress, and temperature distribution in a rotating generalized thermoelastic medium with a hydrostatic initial stress by applying Laplace and Fourier transforms subjected to ramp-type heating and loading. Such types of problems in the rotating medium are very important in many dynamical systems. Some particular cases are also derived from the present investigation.

2 Formulation of the Problem

We consider a homogeneous generalized thermoelastic half-space with a hydrostatic initial stress rotating uniformly with an angular velocity $\vec{\Omega} = \Omega\hat{n}$, where $\hat{n}$ is a unit vector representing the direction of the axis of rotation. A ramp-type source acting at the plane surface of a generalized thermoelastic half-space with a hydrostatic initial stress is considered.

3 Basic Equations and Their Solutions

The constitutive relations and field equations in generalized linear thermoelasticity with a initial hydrostatic stress and without body forces and heat sources are given by Lord and Shulman [2], Green and Lindsay [5], and Montanaro [20] as

$$t_{ij} = -p\left(\delta_{ij} + \omega_{ij}\right) + 2\mu e_{ij} + \lambda e \delta_{ij} - \upsilon \left(1 + \vartheta_0 \frac{\partial}{\partial t}\right) T, \tag{1}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{2}$$

$$\omega_{ij} = \frac{1}{2}\left(u_{j,i} - u_{i,j}\right), \tag{3}$$

$$\left(\mu - \frac{p}{2}\right) u_{i,kk} + \left(\lambda + \mu + \frac{p}{2}\right) u_{k,ik} - \upsilon \left(1 + \vartheta_0 \frac{\partial}{\partial t}\right) T_{,i} = \rho \ddot{u}_i. \tag{4}$$

The heat conduction equation is given by

$$K^* \left(n^* + t_1 \frac{\partial}{\partial t}\right) T_{,ii} = \rho C_E \left(n_1 \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T + \upsilon T_0 \left(n_1 \frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2}\right) u_{i,i}. \tag{5}$$

The equation of motion in a rotating frame of reference has two additional terms: (i) centripetal acceleration $\vec{\Omega} \times \left(\vec{\Omega} \times \vec{u}\right)$ due to time varying motion only and (ii) coriolis acceleration $2\vec{\Omega} \times \dot{\vec{u}}$. So, Eq. 4 can be modified in the rotating medium as

$$\left(\mu - \frac{p}{2}\right) u_{i,kk} + \left(\lambda + \mu + \frac{p}{2}\right) u_{k,ik} - \upsilon \left(1 + \vartheta_0 \frac{\partial}{\partial t}\right) T_{,i} = \rho \left[\ddot{u}_i + \left\{\vec{\Omega} \times \left(\vec{\Omega} \times \vec{u}\right)\right\}_i + \left(2\vec{\Omega} \times \dot{\vec{u}}\right)_i\right]. \tag{6}$$

For a two-dimensional problem (xy−plane), all quantities depends only on space coordinates x, y, and time t; hence, the components of displacement and angular velocity are

$$\vec{u} = (u_1, u_2, 0)\,,\, \vec{\Omega} = (0, 0, \Omega)\,. \tag{7}$$

Using Eq. 7 in Eq. 6, the equations of motion in two dimensions are given by

$$(\lambda + 2\mu)\frac{\partial^2 u_1}{\partial x^2} + \left(\lambda + \mu + \frac{p}{2}\right)\frac{\partial^2 u_2}{\partial x \partial y} + \left(\mu - \frac{p}{2}\right)\frac{\partial^2 u_1}{\partial y^2} - \upsilon\left(1 + \vartheta_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}$$
$$= \rho\left[\frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 - 2\Omega\,\frac{\partial u_2}{\partial t}\right], \tag{8}$$

$$(\lambda + 2\mu)\frac{\partial^2 u_2}{\partial y^2} + \left(\lambda + \mu + \frac{p}{2}\right)\frac{\partial^2 u_1}{\partial x \partial y} + \left(\mu - \frac{p}{2}\right)\frac{\partial^2 u_2}{\partial x^2} - \upsilon\left(1 + \vartheta_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}$$
$$= \rho\left[\frac{\partial^2 u_2}{\partial t^2} - \Omega^2 u_2 + 2\Omega\frac{\partial u_1}{\partial t}\right]. \tag{9}$$

Introducing dimensionless variables defined by

$$x_i' = \frac{\omega^*}{c_0}x_i,\, u_i' = \frac{\rho c_0 \omega^*}{\upsilon T_0}u_i,\, t' = \omega^* t,\, \tau_0' = \omega^*\tau_0,\, \vartheta_0' = \omega^*\vartheta_0,\, T' = \frac{T}{T_0},$$
$$t_{ij}' = \frac{t_{ij}}{\upsilon T_0},\, \Omega' = \frac{\Omega}{\omega^*},\, p' = \frac{p}{\upsilon T_0}, \tag{10}$$

where $\omega^* = \rho C_E c_0^2/K^*$ and $\rho c_0^2 = \lambda + 2\mu$ in Eqs. 5, 8, and 9, we obtain the equations of motion in dimensionless form.

We define displacement potentials ϕ and ψ, which are related to displacement components u_1 and u_2 as

$$u_1 = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial y}, \quad u_2 = \frac{\partial\phi}{\partial y} + \frac{\partial\psi}{\partial x}, \tag{11}$$

in the resulting dimensionless equations, and then applying the Laplace and Fourier transform defined by

$$\bar{f}\,(x, y, s) = \int_0^\infty f\,(x, y, t)\mathrm{e}^{-st}\mathrm{d}t, \tag{12}$$

$$\tilde{f}(\xi, y, s) = \int_{-\infty}^{\infty} \bar{f}\,(x,\, y,\, s)\; \mathrm{e}^{\mathrm{i}\xi x}\mathrm{d}x, \tag{13}$$

we get, (after neglecting the primes),

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}y^2}-\xi^2+\Omega^2-s^2\right]\tilde{\phi}+2\Omega s\tilde{\psi}-(1+\vartheta_0 s)\,\tilde{T}=0, \tag{14}$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}y^2}-\xi^2+a_1\Omega^2-a_1s^2\right]\tilde{\psi}-2\Omega a_1 s\tilde{\phi}=0, \tag{15}$$

$$\left[\frac{\mathrm{d}}{\mathrm{d}y^2}-\xi^2-s\left(\frac{n_1+\tau_0 s}{n^{\bullet}+t_1 s}\right)\right]\tilde{T}-\frac{n_1+\tau_0 n_0 s}{n^{\bullet}+t_1 s}\,(\varepsilon s)\left[\frac{\mathrm{d}}{\mathrm{d}y^2}-\xi^2\right]\tilde{\phi}=0. \tag{16}$$

Eliminating $\tilde{\phi}$ and $\tilde{\psi}$ from Eqs. 14–16, we obtain

$$\left[\nabla^6-A\nabla^4+B\nabla^2-C\right]\tilde{\phi}=0, \tag{17}$$

where

$$\begin{aligned}
&\nabla=\frac{\mathrm{d}}{\mathrm{d}y},\quad a_1=\frac{\rho c_0^2}{\mu-\frac{vT_0p}{2}},\quad \in=\frac{\upsilon^2T_0}{\rho K^*\omega^*},\quad e_1=\xi^2+s\left(\frac{n_1+\tau_0 s}{n^{\bullet}+t_1 s}\right),\\
&e_2=\xi^2-\Omega^2+s^2,\quad e_3=\varepsilon s\,(1+\vartheta_0 s)\left(\frac{n_1+n_0\tau_0 s}{n^{\bullet}+t_1 s}\right),\\
&e_4=\xi^2-a_1\Omega^2+a_1 s^2,\\
&A=e_1+e_2+e_3+e_4,\\
&B=e_4\,(e_1+e_2+e_3)+e_1e_2+e_3\xi^2+4a_1\Omega^2 s^2,\\
&C=e_4\left(e_1e_2+e_3\xi^2\right)+4e_1a_1s^2\Omega^2.
\end{aligned} \tag{18}$$

The solutions of Eq. 17 satisfying the radiation conditions that $\tilde{\phi},\tilde{\psi},\tilde{T}\to 0$ as $y\to\infty$ are

$$\tilde{\phi}=D_1\mathrm{e}^{-\phi_1 y}+D_2\mathrm{e}^{-\phi_2 y}+D_3\mathrm{e}^{-\phi_3 y}, \tag{19}$$

$$\tilde{\psi}=a_1^*D_1\mathrm{e}^{-\phi_1 y}+a_2^*D_2\mathrm{e}^{-\phi_2 y}+a_3^*D_3\mathrm{e}^{-\phi_3 y}, \tag{20}$$

$$\tilde{T}=b_1^*D_1e^{-\phi_1 y}+b_2^*D_2e^{-\phi_2 y}+b_3^*D_3e^{-\phi_3 y}, \tag{21}$$

where ϕ_i^2 are the roots of the characteristic Eq. 17 and a_i^*, b_i^* are coupling constants defined by

$$a_i^*=\frac{\phi_i^4-(e_1+e_2+e_3)\,\phi_i^2+\left(e_1e_2+e_3\xi^2\right)}{2\Omega s\left(\phi_i^2-e_1\right)}, \tag{22}$$

$$b_i^*=\varepsilon s\left(\frac{n_1+n_0\tau_0 s}{n^{\bullet}+t_1 s}\right)\left(\frac{\phi_i^2-\xi^2}{\phi_i^2-e_1}\right),\quad i=1,2,3. \tag{23}$$

4 Boundary Conditions

4.1 Mechanical Force

The boundary conditions at the plane surface $y = 0$ are

$$\begin{aligned} &\text{(i)} \quad t_{22} = F(x,t) = \begin{bmatrix} 0 & t \leq 0 \\ \sigma_1 \frac{t}{t_0} & 0 < t \leq t_0 \\ \sigma_1 & t > t_0 \end{bmatrix} \\ &\text{(ii)} \quad t_{21} = 0, \\ &\text{(iii)} \quad T = 0. \end{aligned} \tag{24}$$

Using Eqs. 1, 7, 10, and 11 in the boundary conditions (Eq. 24), we obtain the boundary conditions in dimensionless form. On suppressing the primes and applying the Laplace and Fourier transforms defined by Eqs. 12 and 13 on the dimensionless boundary conditions and using Eqs. 19–21 in the resulting transformed boundary conditions, we get the transformed expressions for displacement, force stress, and temperature distribution in a rotating generalized thermoelastic medium with a hydrostatic initial stress as

$$\tilde{u}_1 = \frac{\left(\sum_{m=1}^{3} b_m \Delta_m \mathrm{e}^{-\phi_m y}\right)}{\Delta}, \tag{25}$$

$$\tilde{u}_2 = \frac{\left(\sum_{m=1}^{3} l_m \Delta_m \mathrm{e}^{-\phi_m y}\right)}{\Delta}, \tag{26}$$

$$\tilde{t}_{21} = \frac{\left(\sum_{m=1}^{3} q_m \Delta_m \mathrm{e}^{-\phi_m y}\right)}{\Delta}, \tag{27}$$

$$\tilde{t}_{22} = \frac{\left(\sum_{m=1}^{3} r_m \Delta_m \mathrm{e}^{-\phi_m y}\right)}{\Delta}, \tag{28}$$

$$\tilde{T} = \frac{\left(\sum_{m=1}^{3} b_m^{\bullet} \Delta_m \mathrm{e}^{-\phi_m y}\right)}{\Delta}, \tag{29}$$

where

$$\Delta = \left[\frac{r_1 \Delta_1 + r_2 \Delta_2 + r_3 \Delta_3}{p - \tilde{F}(\xi, s)}\right], \Delta_1 = \left(p - \tilde{F}(\xi, s)\right)\left[q_2 b_3^{\bullet} - b_2^{\bullet} q_3\right],$$

$$\Delta_2 = -\left(p - \tilde{F}(\xi, s)\right)\left[q_1 b_3^{\bullet} - b_1^{\bullet} q_3\right],$$

$$\Delta_3 = \left(p - \tilde{F}(\xi, s)\right)\left[q_1 b_2^{\bullet} - b_1^{\bullet} q_2\right],$$

$$q_i = -\left[\frac{\left(\mu - \frac{\upsilon T_0 p}{2}\right) a_i^{\bullet} \phi_i^2}{\rho c_0^2} + \frac{\left(\mu + \frac{\upsilon T_0 p}{2}\right) a_i^{\bullet} \xi^2}{\rho c_0^2} - \frac{2 i \xi \mu \phi_i}{\rho c_0^2}\right]$$

$$r_i = \phi_i^2 - \frac{\lambda\xi^2}{\rho c_0^2} + \frac{2i\xi\mu a_i^\bullet \phi_i}{\rho c_0^2} - (1+\vartheta_0 s)\, b_i^\bullet, \quad i = m = 1, 2, 3.$$

$$b_i = a_i^\bullet \phi_i - i\xi, l_i = -\left(a_i^\bullet i\xi + \phi_i\right), \tilde{F}(\xi, s) = \frac{\sigma_1}{p^2 t_0}\left(1 - \mathrm{e}^{-st_0}\right). \tag{30}$$

4.2 Thermal Source

The boundary conditions at the plane surface $y = 0$ subjected to a thermal source are

(i) $t_{22} = 0$,
(ii) $t_{21} = 0$,
(iii) $T = H(x, t) = \begin{bmatrix} 0 & t \le 0 \\ T_1 \frac{t}{t_0} & 0 < t \le t_0 \\ T_1 & t > t_0 \end{bmatrix}.$

For the case of a thermal source, the expressions for displacement, force stress, and temperature distribution are obtained by replacing Δ_m by $\Delta_m^\bullet$ in Eqs. 25–29, where

$$\begin{aligned} \Delta_1^\bullet &= p\left(q_2 b_3^\bullet - b_2^\bullet q_3\right) + \tilde{H}(\xi, s)\,(r_2 q_3 - q_2 r_3), \\ \Delta_2^\bullet &= -p\left(q_1 b_3^\bullet - b_1^\bullet q_3\right) - \tilde{H}(\xi, s)\,(r_1 q_3 - q_1 r_3), \\ \Delta_3^\bullet &= p\left(q_1 b_2^\bullet - b_1^\bullet q_2\right) + \tilde{H}(\xi, s)\,(r_1 q_2 - q_1 r_2), \\ \tilde{H}(\xi, s) &= \frac{T_1}{s^2 t_0}\left(1 - \mathrm{e}^{-st_0}\right). \end{aligned} \tag{31}$$

5 Particular Cases

5.1 Neglecting the Angular Velocity

Neglecting the angular velocity (i.e., $\vec{\Omega} = 0$), we obtain transformed components of displacement, stress forces, and temperature distribution in a non-rotating generalized thermoelastic medium with a hydrostatic initial stress as

$$\tilde{u}_1 = \frac{\left(\sum_{m=1}^{3} b'_m \Delta_m^{(1)} \mathrm{e}^{-\phi'_m y}\right)}{\Delta^{(1)}}, \tag{32}$$

$$\tilde{u}_2 = \frac{\left(\sum_{m=1}^{3} l'_m \Delta_m^{(1)} \mathrm{e}^{-\phi'_m y}\right)}{\Delta^{(1)}}, \tag{33}$$

$$\tilde{t}_{21} = \frac{\left(\sum_{m=1}^{3} q'_m \Delta_m^{(1)} \mathrm{e}^{-\phi'_m y}\right)}{\Delta^{(1)}}, \tag{34}$$

$$\tilde{t}_{22} = \frac{\left(\sum_{m=1}^{3} r'_m \Delta_m^{(1)} \mathrm{e}^{-\phi'_m y}\right)}{\Delta^{(1)}}, \tag{35}$$

$$\tilde{T} = \frac{\left(\sum_{m=1}^{3} b'^{\bullet}_m \Delta_m^{(1)} \mathrm{e}^{-\phi'_m y}\right)}{\Delta^{(1)}}, \tag{36}$$

where

$$\begin{aligned}
&\Delta^{(1)} = r'_3 \frac{\Delta_3^{(1)}}{p - \tilde{F}(\xi, s)} - q'_3 \left(r'_1 b'^{\bullet}_2 - b'^{\bullet}_1 r'_2\right), \Delta_1^{(1)} = -\left[p - \tilde{F}(\xi, s)\right] b'^{\bullet}_2 q'_3, \\
&\Delta_2^{(1)} = \left[p - \tilde{F}(\xi, s)\right] b'^{\bullet}_1 q'_3, \Delta_3^{(1)} = \left[p - \tilde{F}(\xi, s)\right] \left[q'_1 b'^{\bullet}_2 - b'^{\bullet}_1 q'_2\right], \\
&r'_{1,2} = \phi'^2_{1,2} - \frac{\lambda \xi^2}{\rho c_0^2} - (1 + \vartheta_0 p)\, b'^{\bullet}_{1,2}, r'_3 = \frac{2\mu i \xi \phi'_3}{\rho c_0^2}, b'_{1,2} = -i\xi, b'_3 = \phi'_3, \\
&q'_{1,2} = \frac{2i\xi\mu\phi'_{1,2}}{\rho c_0^2}, q''_3 = -\left[\frac{\left(\mu - \frac{\upsilon T_0 p}{2}\right)\phi'^2_3}{\rho c_0^2} + \frac{\left(\mu + \frac{\upsilon T_0 p}{2}\right)\xi^2}{\rho c_0^2}\right], \\
&l'_{1,2} = -\phi'_{1,2}, l'_3 = -i\xi, \\
&b'^{\bullet}_{1,2} = \frac{\phi'^2_{1,2} - e'_2}{1 + \vartheta_0 s}, \quad A_1 = e_1 + e'_2 + e_3, B_1 = \left(e_1 e'_2 + e_3 \xi^2\right), \\
&\phi'_{1,2} = \frac{A_1 \pm \sqrt{A_1 - 4B_1}}{2}, \phi'^2_3 = \xi^2 + \frac{\rho c_0^2 s^2}{\left(\mu - \frac{\upsilon T_0 p}{2}\right)}, e'_2 = \xi^2 + s^2.
\end{aligned} \tag{37}$$

5.1.1 Thermal Source

In this case, the expressions for displacement, force stress, and temperature distribution are reduced by replacing $\Delta_m^{(1)}$ by $\Delta_m^{\bullet(1)}$ in Eqs. 32– 36, where

$$\begin{aligned}
&\Delta_1^{\bullet(1)} = -p b'^{\bullet}_2 q'_3 + \tilde{H}(\xi, s)\left(r'_2 q'_3 - q'_2 r'_3\right), \Delta_2^{\bullet(1)} = p b'^{\bullet}_1 q'_3 - \tilde{H}(\xi, s)\left(r'_1 q'_3 - q'_1 r'_3\right), \\
&\Delta_3^{\bullet(1)} = p\left(q'_1 b'^{\bullet}_2 - b'^{\bullet}_1 q'_2\right) + \tilde{H}(\xi, s)\left(r'_1 q'_2 - q'_1 r'_2\right).
\end{aligned}$$

5.2 Neglecting Angular Velocity and Hydrostatic Initial Stress

Neglecting both angular velocity and hydrostatic initial stress (i.e., $\vec{\Omega} = p = 0$), we get the expressions for displacement, force stresses, and temperature distribution in a non-rotating thermoelastic medium as

$$\tilde{u}_1 = \frac{\left(\sum_{m=1}^{3} b''_m \Delta_m^{(2)} \mathrm{e}^{-\phi''_m y}\right)}{\Delta^{(2)}}, \tag{38}$$

$$\tilde{u}_2 = \frac{\left(\sum_{m=1}^{3} l''_m \Delta_m^{(2)} \mathrm{e}^{-\phi''_m y}\right)}{\Delta^{(2)}}, \tag{39}$$

$$\tilde{t}_{21} = \frac{\left(\sum_{m=1}^{3} q''_m \Delta_m^{(2)} \mathrm{e}^{-\phi''_m y}\right)}{\Delta^{(2)}}, \tag{40}$$

$$\tilde{t}_{22} = \frac{\left(\sum_{m=1}^{3} r''_m \Delta_m^{(2)} \mathrm{e}^{-\phi''_m y}\right)}{\Delta^{(2)}}, \tag{41}$$

$$\tilde{T} = \frac{\left(\sum_{m=1}^{2} b''^{\bullet}_m \Delta_m^{(2)} \mathrm{e}^{-\phi''_m y}\right)}{\Delta^{(2)}}, \tag{42}$$

where

$$\begin{aligned}
&\Delta^{(2)} = -\left[\frac{r''_3 \Delta_3^{(2)}}{\tilde{F}(\xi, s)} + q''_3\left(r''_1 b''^{\bullet}_2 - b''^{\bullet}_1 r''_2\right)\right], \ \Delta_1^{(2)} = \tilde{F}(\xi, s)\, b''^{\bullet}_2 q''_3, \\
&\Delta_2^{(2)} = -\frac{\Delta_1^{(2)} b''^{\bullet}_1}{b''^{\bullet}_2}, \ \Delta_3^{(2)} = -\tilde{F}(\xi, s)\left[q''_1 b''^{\bullet}_2 - b''^{\bullet}_1 q''_2\right], \\
&A_2 = e_1 + e'_2 + e_3, B_2 = e_1 e'_2 + e_3 \xi^2, \\
&\phi''^2_{1,2} = \frac{A_2 \pm \sqrt{A_2^2 - 4B_2}}{2}, \quad \phi''^2_3 = \xi^2 + \frac{s^2 \rho c_0^2}{\mu}, \quad b''^{\bullet}_{1,2} = \frac{\phi''^2_{1,2} - e'_2}{1 + \vartheta_0 s}, \\
&r''_{1,2} = \phi''^2_{1,2} - \frac{\lambda \xi^2}{\rho c_0^2} - (1 + \vartheta_0 p)\, b''^{\bullet}_{1,2}, \quad r''_3 = \frac{2\mu i \xi \phi''_3}{\rho c_0^2}, \\
&q''_{1,2} = \frac{2i\xi\mu\phi''_{1,2}}{\rho c_0^2}, \quad s''_3 = -\frac{\left(\phi''^2_3 + \xi^2\right)\mu}{\rho c_0^2}, \\
&b''_{1,2} = -i\xi, \quad b''_3 = \phi''_3, \quad l''_{1,2} = -\phi''_{1,2}, \quad l''_3 = -i\xi.
\end{aligned} \tag{43}$$

5.2.1 Thermal Source

We obtained the expressions for displacement, force stress, and temperature distribution by replacing $\Delta_m^{(2)}$ by $\Delta_m^{\bullet(2)}$ in Eqs. 38– 42 for ramp-type heating, where

$$\begin{aligned}
&\Delta_1^{\bullet(2)} = \tilde{H}(\xi, s)\left(r''_2 q''_3 - q''_2 r''_3\right), \Delta_2^{\bullet(2)} = -\tilde{H}(\xi, s)\left(r''_1 q''_3 - q''_1 r''_3\right), \\
&\Delta_3^{\bullet(2)} = \tilde{H}(\xi, s)\left(r''_1 q''_2 - q''_1 r''_2\right).
\end{aligned}$$

5.3 Neglecting Hydrostatic Initial Stress

Neglecting hydrostatic initial stress (i.e., $p = 0$), we obtain transformed components of displacement, stress forces, and temperature distribution in a rotating generalized thermoelastic medium.

6 Numerical Results

With a view to illustrating the analytical procedure presented earlier, we now consider a numerical example for which computational results are given. The results depict the variations of temperature, displacement, and stress fields in the context of the G–L theory. For this purpose, we take the following values of physical constants as

$$
\begin{aligned}
&E = 36.9 \times 10^{10}\,\mathrm{N \cdot m^{-2}}, \sigma = 0.33, \rho = 2.7 \times 10^{3}\,\mathrm{kg \cdot m^{-3}},\\
&C_E = 0.9878 \times 10^{3}\,\mathrm{J \cdot kg^{-1} \cdot {}^{\circ}C^{-1}}\\
&K^{*} = 2.059 \times 10^{3}\,\mathrm{J \cdot m^{-1} \cdot {}^{\circ}C^{-1}}, \nu = \frac{\alpha}{\rho K_T}, \alpha = 0.01, K_T = 0.5, T_0 = 20\,{}^{\circ}\mathrm{C}\\
&\mu = \frac{E}{2\eta\,(1+\sigma)}, \lambda = \frac{E\sigma}{\eta\,(1+\sigma)\,(1-2\sigma)}.
\end{aligned}
$$

$\eta = 1$ corresponds to an isotropic elastic medium.

The computations are carried out on the surface $y = 1.0$ at $t = 1.0$ for two values of ramping parameters t_0(0.1 and 0.5). The graphical results for the normal displacement u_2, the normal force stress t_{22}, and temperature distribution T are shown in Figs. 1 to 12 with $\Omega = 0.5$ and $p = 1.0$ for a

(a) generalized thermoelastic medium with a hydrostatic initial stress and rotation (GTESHR) by solid line (———),
(b) generalized thermoelastic medium with a hydrostatic initial stress and without rotation (GTESHWR) by dashed line (-------),
(c) generalized thermoelastic medium with rotation and without a hydrostatic initial stress (GTESR) by solid line with centered symbol (*—*—*), and
(d) generalized thermoelastic solid without rotation and without a hydrostatic initial stress (GTESWR) by dashed line with centered symbol (*---*---*).

These graphical results represent the solutions obtained by using the generalized theory with two relaxation times (G–L-theory by taking $\tau_0 = 0.03$, $\vartheta_0 = 0.05$.)

7 Inversion of the Transform

The transformed displacements, microrotation, and stresses are functions of y, the parameters of Laplace and Fourier transforms s and ξ, respectively, and hence are of the form $\tilde{f}\,(\xi, y, s)$. To get the function in the physical domain, first we invert the Fourier transform and then the Laplace transform by using the method applied by Sharma and Kumar [44].

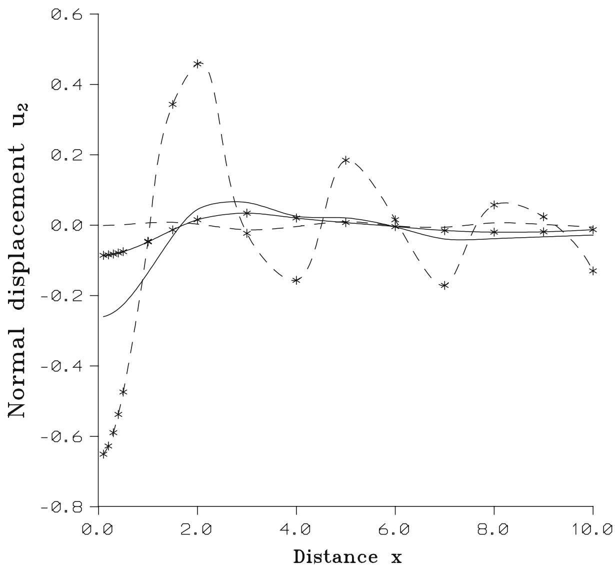

Fig. 1 Variation of normal displacement u_2 with horizontal distance x for a mechanical force and $t_0 = 0.1$: (————) GTESHR—generalized thermoelastic medium with a hydrostatic initial stress and rotation, (∗——∗——∗) GTESR—generalized thermoelastic medium with rotation and without a hydrostatic initial stress, (- - - - - - -) GTESHWR—generalized thermoelastic medium with a hydrostatic initial stress and without rotation, and (∗- - -∗- - -∗) GTESWR—generalized thermoelastic solid without rotation and without a hydrostatic initial stress

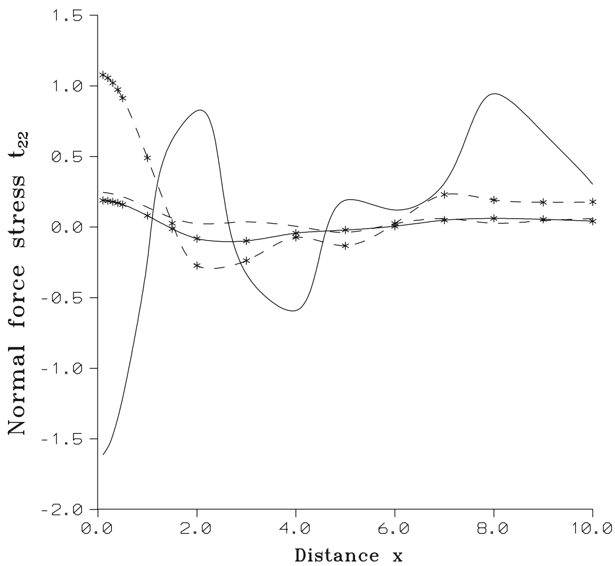

Fig. 2 Variation of normal force stress t_{22} with horizontal distance x for a mechanical force and $t_0 = 0.1$: (————) GTESHR, (∗——∗——∗) GTESR, (- - - - - - -) GTESHWR, and (∗- - -∗- - -∗) GTESWR [see Fig. 1 for explanation of symbols]

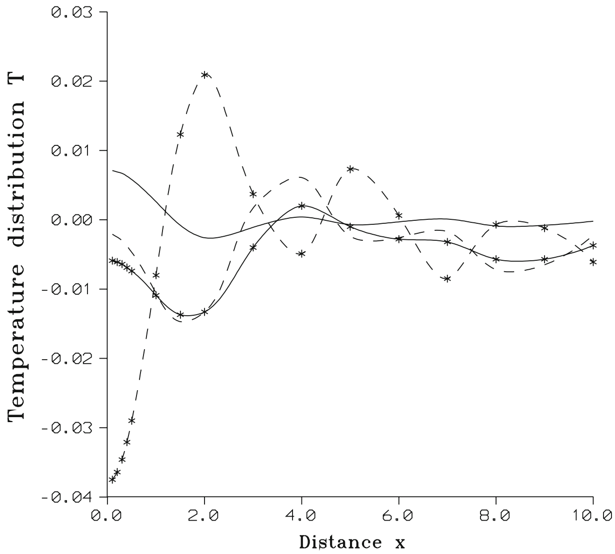

Fig. 3 Variation of temperature distribution T with horizontal distance x for a mechanical force and $t_0 = 0.1$: (———) GTESHR, (∗—∗—∗) GTESR, (- - - - -) GTESHWR, and (∗- - -∗- - -∗) GTESWR [see Fig. 1 for explanation of symbols]

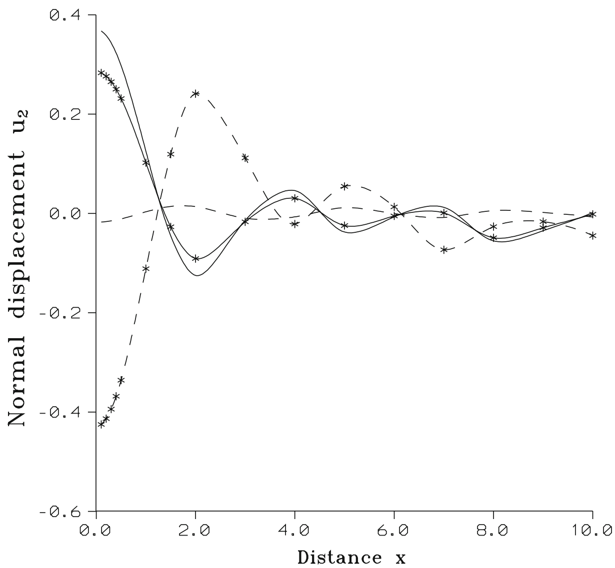

Fig. 4 Variation of normal displacement u_2 with horizontal distance x for a thermal source and $t_0 = 0.1$: (———) GTESHR, (∗—∗—∗) GTESR, (- - - - -) GTESHWR, and (∗- - -∗- - -∗) GTESWR [see Fig. 1 for explanation of symbols]

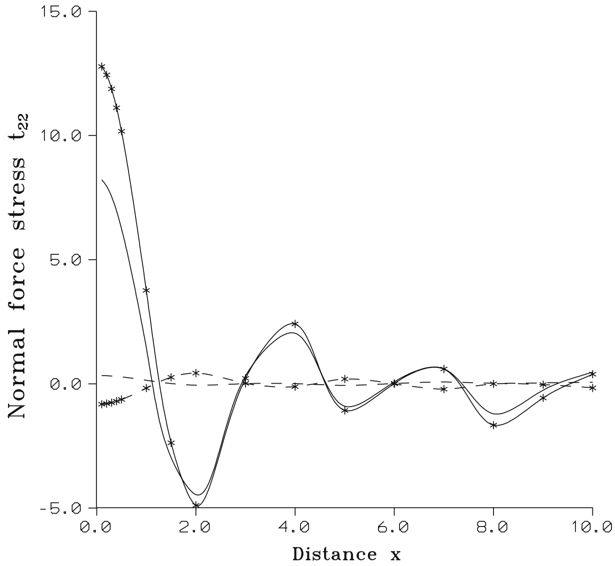

Fig. 5 Variation of normal force stress $t_{22}$with horizontal distance x for a thermal source and $t_0 = 0.1$: (———) GTESHR, (∗—∗—∗) GTESR, (- - - - -) GTESHWR, and (∗- - -∗- - -∗) GTESWR [see Fig. 1 for explanation of symbols]

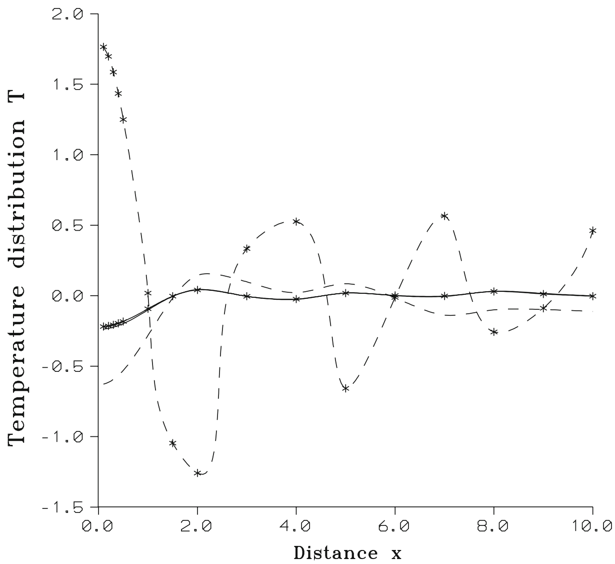

Fig. 6 Variation of temperature distribution T with horizontal distance x for a thermal source and $t_0 = 0.1$: (———) GTESHR, (∗—∗—∗) GTESR, (- - - - -) GTESHWR, and (∗- - -∗- - -∗) GTESWR [see Fig. 1 for explanation of symbols]

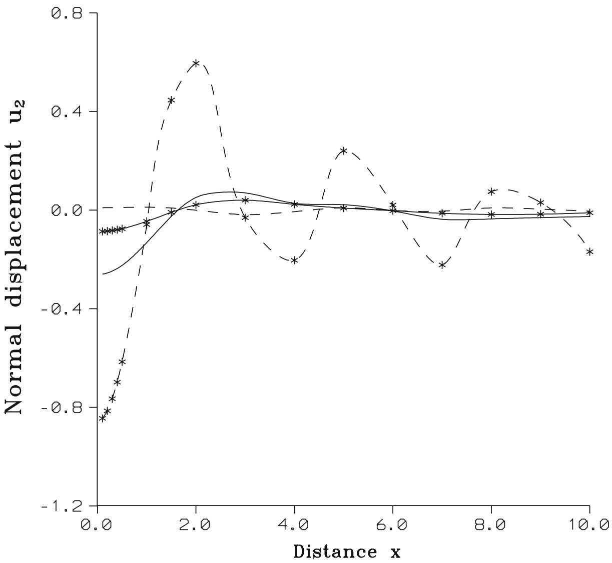

Fig. 7 Variation of normal displacement u_2 with horizontal distance x for a mechanical force and $t_0 = 0.5$: (———) GTESHR, (∗—∗—∗) GTESR, (- - - - - -) GTESHWR, and (∗- - -∗- - -∗) GTESWR [see Fig. 1 for explanation of symbols]

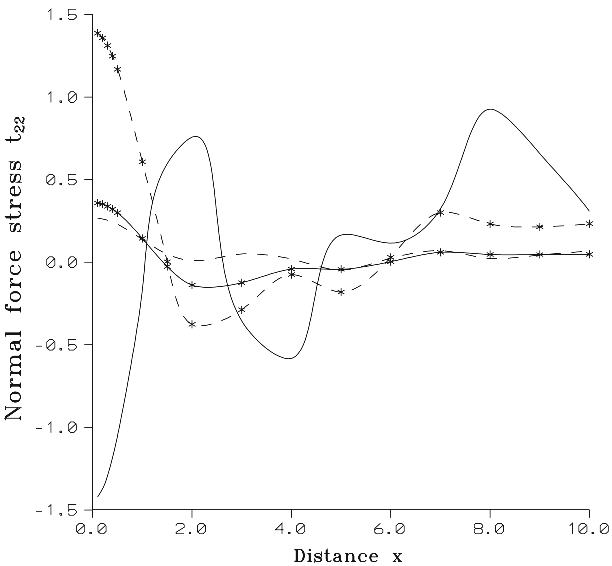

Fig. 8 Variation of normal force stress t_{22} with horizontal distance x for a mechanical force and $t_0 = 0.5$: (———) GTESHR, (∗—∗—∗) GTESR, (- - - - - -) GTESHWR, and (∗- - -∗- - -∗) GTESWR [see Fig. 1 for explanation of symbols]

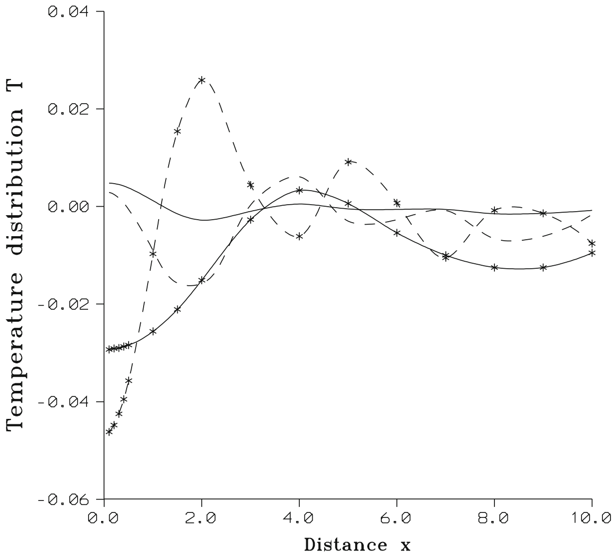

Fig. 9 Variation of temperature distribution T with horizontal distance x for a mechanical force and $t_0 = 0.5$: (————) GTESHR, (∗——∗——∗) GTESR, (- - - - - -) GTESHWR, and (∗- - -∗- - -∗) GTESWR [see Fig. 1 for explanation of symbols]

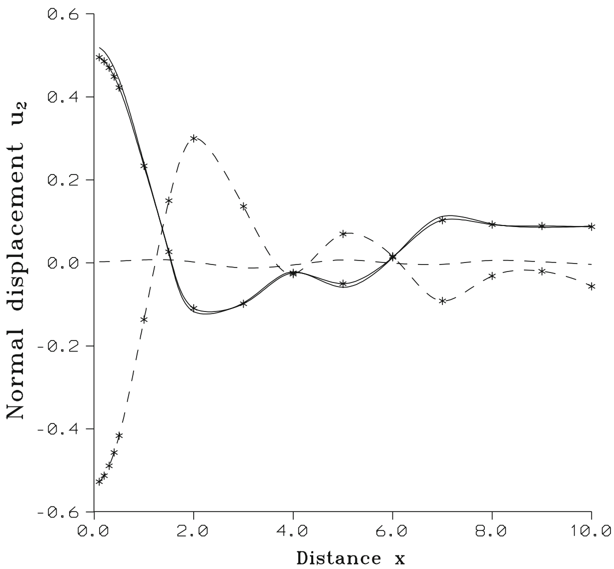

Fig. 10 Variation of normal displacement u_2 with horizontal distance x for a thermal source and $t_0 = 0.5$: (————) GTESHR, (∗——∗——∗) GTESR, (- - - - - -) GTESHWR, and (∗- - -∗- - -∗) GTESWR [see Fig. 1 for explanation of symbols]

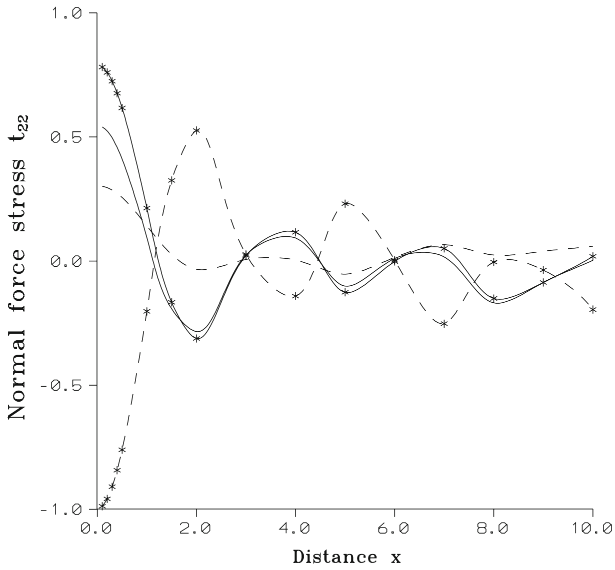

Fig. 11 Variation of normal force stress $t_{22}$with horizontal distance x for a thermal source and $t_0 = 0.5$: (————) GTESHR, (∗—∗—∗) GTESR, (- - - - - -) GTESHWR, and (∗- - -∗- - -∗) GTESWR [see Fig. 1 for explanation of symbols]

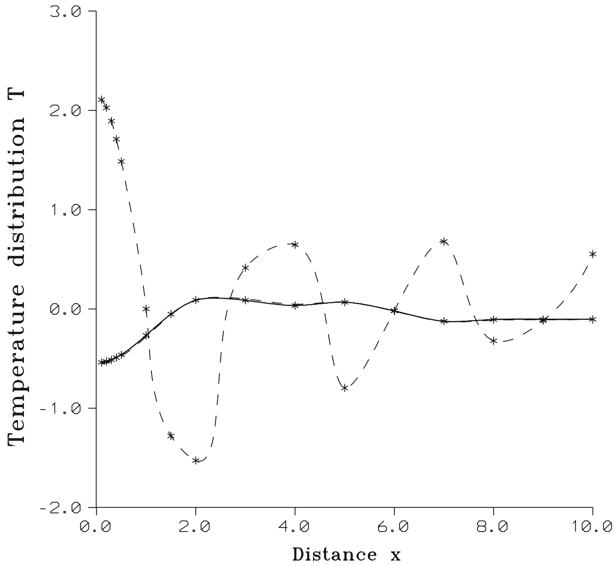

Fig. 12 Variation of temperature distribution T with horizontal distance x for a thermal source and $t_0 = 0.5$: (————) GTESHR, (∗—∗—∗) GTESR, (- - - - - -) GTESHWR, and (∗- - -∗- - -∗) GTESWR [see Fig. 1 for explanation of symbols]

8 Special Cases of Thermoelasticity Theory

8.1 Equations of Coupled thermoelasticity

The equations of the coupled thermoelasticity (C-T theory) for a rotating media are obtained when

$$n^* = n_1 = 1, t_1 = \tau_0 = \vartheta_0 = 0. \tag{44}$$

8.2 Lord–Shulman Theory

For the Lord–Shulman (L–S theory),

$$n^* = n_1 = n_0 = 1, t_1 = \vartheta_0 = 0, \tau_0 > 0. \tag{45}$$

8.3 Green–Lindsay Theory

For the Green–Lindsay (G–L theory),

$$n^* = n_1 = 1, n^* = 0, t_1 = 0, \vartheta_0 \geq \tau_0 > 0, \tag{46}$$

where ϑ_0, τ_0 are two relaxation times.

8.4 Equations of Generalized Thermoelasticity

The equations of generalized thermoelasticity for a rotating medium, without energy dissipation (the linearized GN theory of type II) are obtained when

$$n^* > 0, n_1 = 0, n_0 = 1,\ \ t_1 = \vartheta_0 = 0, \tau_0 = 1, \tag{47}$$

Equations 1 and 4 are the same, and Eq. 5 takes the form,

$$K^* \nabla^2 T = \rho C_E \frac{\partial^2 T}{\partial t^2} + \upsilon T_0 \frac{\partial^2 e}{\partial t^2}, \tag{48}$$

where n^* is a constant, which has the dimension of $\left[\frac{1}{s}\right]$, and $n^* k^* = K' = C_E \left(\lambda + 2\mu\right)/4$ is a characteristic constant of this theory.

9 Discussion

9.1 Mechanical Force with Ramping Parameter $t_0 = 0.1$

The values of the normal displacement in the case of a generalized thermoelastic medium are highly oscillatory in nature when both the hydrostatic initial stress and angular velocity are neglected. For the case of a hydrostatic initial stress and (or) angular velocity, the values of the normal displacement lie in a very short range. These variations of normal displacement are shown in Fig. 1.

It is observed from Fig. 2 that the values of the normal force stress also lie in a short range if either the hydrostatic initial stress or angular velocity is neglected. However, for a thermoelastic medium with a hydrostatic initial stress and angular velocity, these variations are oscillatory to a significant effect. The variations of the temperature distribution for the medium (in the absence of a hydrostatic initial stress or angular velocity) are similar in nature with oscillatory behavior. Also, the variations are oscillatory with a decreasing magnitude when the medium is not under the effect of a hydrostatic initial stress and rotation. These variations of the temperature distribution are shown in Fig. 3.

9.2 Thermal Source with Ramping Parameter $t_0 = 0.1$

When a thermal source is acting on the surface of a generalized thermoelastic medium, the variations of the normal displacement and normal force stress are more uniform in nature as compared to the variations obtained on application of a mechanical force. We may observe from Figs. 4 and 5 that the variations of both these quantities are quite similar in nature when the medium is rotating with some angular velocity (with or without a hydrostatic initial stress). Similar to the discussions given above, the variations of the temperature distribution depicted in Fig. 6 are almost identical for a rotating generalized thermoelastic medium.

On changing the values of the ramping parameter t_0, we observe similar behavior of all the quantities with some difference in magnitude. We have shown these variations in Figs. 7 to 12 by taking the value of the ramping parameter $t_0 = 0.5$.

10 Conclusion

A significant effect of the hydrostatic initial stress and angular velocity is observed on all the quantities. Also, the nature of the source (mechanical and thermal) affects the nature of all quantities. The body is deformed to a large extent when a thermal source is applied on the surface of the medium, since all the quantities have been de-magnified by a factor (10^3 to 10^4).

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